G01GRAVITATIONAL FIELDS, ORBITS, & KEPLER'S LAWS

SPH4U

EXPECTATIONS

- analyze the factors affecting the motion of isolated celestial objects and calculate the gravitational potential energy for each system
- analyze isolated planetary and satellite motion, and describe the motion in terms of the forms of energy and energy transformations that occur

EQUATIONS

- Universal Law of Gravitation

$$F_G = \frac{GMm}{r^2}$$

- Gravitational Field Strength

$$g = \frac{GM}{r^2}$$

- Orbital Speed

$$v = \sqrt{\frac{GM}{r}}$$

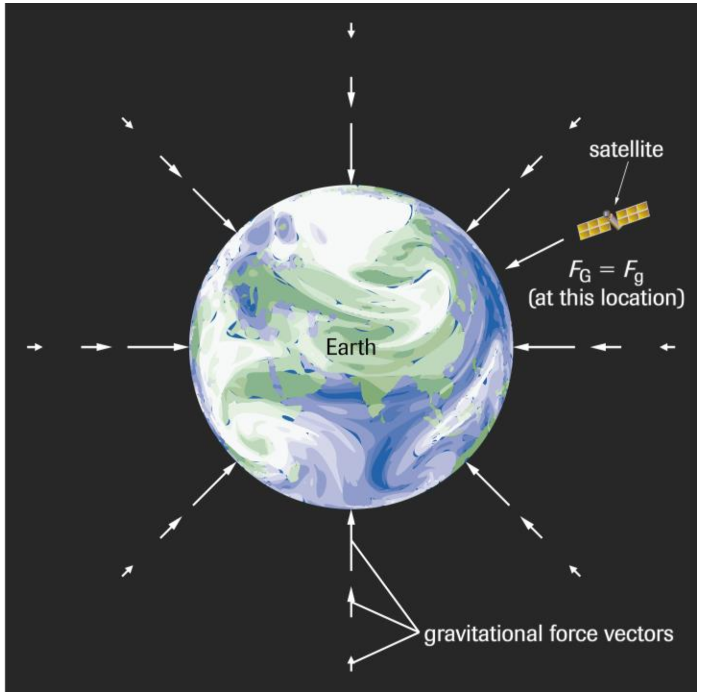
- Kepler's 3rd Law

$$C = \frac{r^3}{T^2}$$

GRAVITATIONAL FIELDS

- **Gravitational Field:** exists in the space surrounding an object in which the force of gravity exists.
  - NOTE: <u>all</u> objects have mass, and therefore generate a gravitational field
- Recall: The Universal Law of Gravitation

$$F_G = \frac{GMm}{r^2}$$



GRAVITATIONAL FIELD STRENGTH

- Let's compare the Universal Law of Gravitation with our simpler form of gravitational force for the surface of Earth

$$F_g = F_G$$

$$mg = \frac{GmM}{r^2}$$

$$\boldsymbol{g = \frac{GM}{r^2}}$$

- g is the gravitational field strength for a given mass M at a point r away from the centre of mass.

PROBLEM 1

Determine the mass of Earth using the magnitude of the gravitational field strength at the surface of the Earth, the distance r between Earth's surface and its centre (6.38×10^6 m), and the universal gravitation constant.

PROBLEM 1 – SOLUTIONS

$g = 9.80$ N/kg $\qquad G = 6.67 \times 10^{-11}$ N·m^2/kg^2

$r = 6.38 \times 10^6$ m $\qquad M = ?$

$$g = \frac{GM}{r^2}$$

$$M = \frac{gr^2}{G}$$

$$= \frac{(9.80\ \text{N/kg})(6.38 \times 10^6\ \text{m})^2}{6.67 \times 10^{-11}\ \text{N·m}^2/\text{kg}^2}$$

$$M = 5.98 \times 10^{24}\ \text{kg}$$

The mass of Earth is 5.98×10^{24} kg.

PROBLEM 2

(a) Calculate the magnitude of the gravitational field strength on the surface of Mars.

(b) What is the ratio of the magnitude of the gravitational field strength on the surface of Mars to that on the surface of Earth?

PROBLEM 2 – SOLUTIONS

Appendix C contains the required data.

(a) $G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$ $r = 3.40 \times 10^6\ \text{m}$

$M = 6.37 \times 10^{23}\ \text{kg}$ $g = ?$

$$g = \frac{GM}{r^2}$$

$$= \frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(6.37 \times 10^{23}\ \text{kg})}{(3.40 \times 10^6\ \text{m})^2}$$

$$g = 3.68\ \text{N/kg}$$

The magnitude of the gravitational field strength on the surface of Mars is 3.68 N/kg.

PROBLEM 2 – SOLUTIONS CONT.

(b) The required ratio is:

$$\frac{g_{\text{Mars}}}{g_{\text{Earth}}} = \frac{3.68\ \text{N/kg}}{9.80\ \text{N/kg}} = 0.375:100$$

The ratio of the magnitudes of the gravitational field strengths is 0.375:100. This means that the gravitational field strength on the surface of Mars is 37.5% of the gravitational field strength on the surface of Earth.

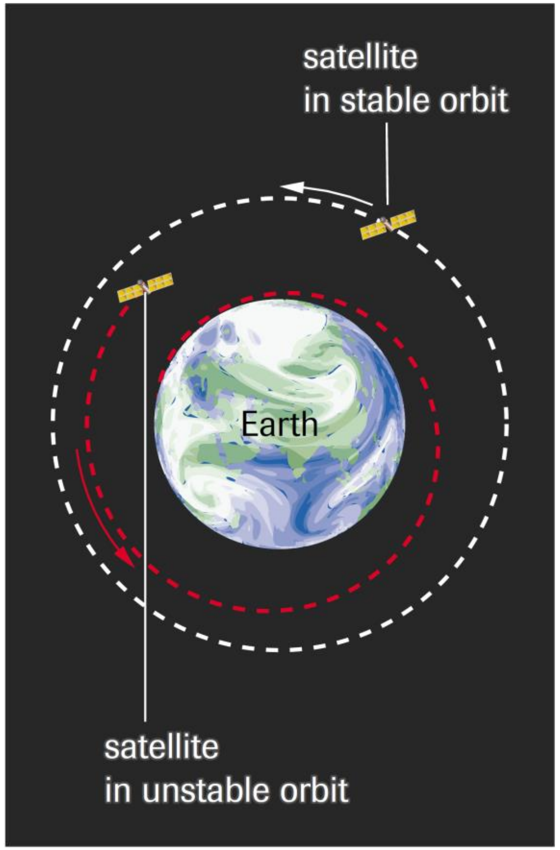
ORBITAL SPEED

- A satellite must travel at a constant speed to maintain a stable orbit; any decrease in speed will cause the orbit to become unstable and the satellite will re-enter the atmosphere.
- Recall: we derived the equation for the speed of a satellite maintaining its orbit around the Earth:

$$v = \sqrt{\frac{Gm_E}{r}}$$

- This can be written in a more generic form for any mass generating a gravitational field

$$v = \sqrt{\frac{GM}{r}}$$



PROBLEM 3

Determine the speeds of the second and third planets from the Sun. Refer to Appendix C for the required data.

$$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$$

$$M_S = 1.99 \times 10^{30}\ \mathrm{kg}$$

$$r_V = 1.08 \times 10^{11}\ \mathrm{m}$$

$$r_E = 1.49 \times 10^{11}\ \mathrm{m}$$

PROBLEM 3 – SOLUTIONS

We will use the subscript V to represent Venus (the second planet), the subscript E to represent Earth (the third planet), and the subscript S to represent the Sun.

$G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$

$M_S = 1.99 \times 10^{30}\ \text{kg}$

$r_V = 1.08 \times 10^{11}\ \text{m}$

$r_E = 1.49 \times 10^{11}\ \text{m}$

$v_V = ?$

$v_E = ?$

$$v_V = \sqrt{\frac{GM_S}{r_V}}$$

$$= \sqrt{\frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99 \times 10^{30}\ \text{kg})}{1.08 \times 10^{11}\ \text{m}}}$$

$$v_V = 3.51 \times 10^4\ \text{m/s}$$

PROBLEM 3 – SOLUTIONS CONT.

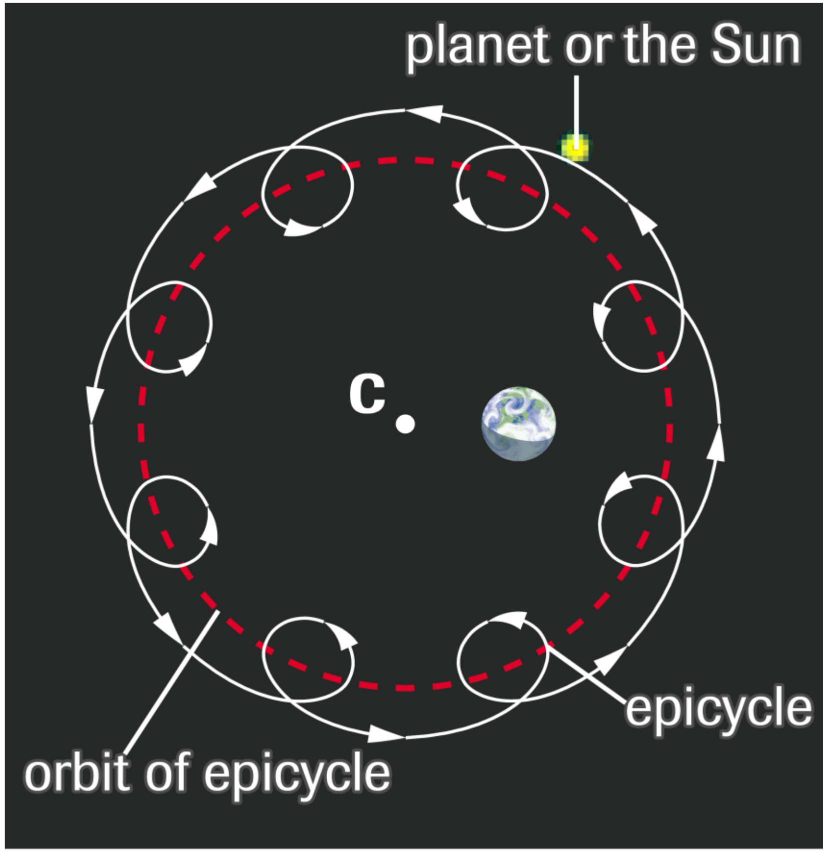
$$v_E = \sqrt{\frac{GM_S}{r_E}}$$

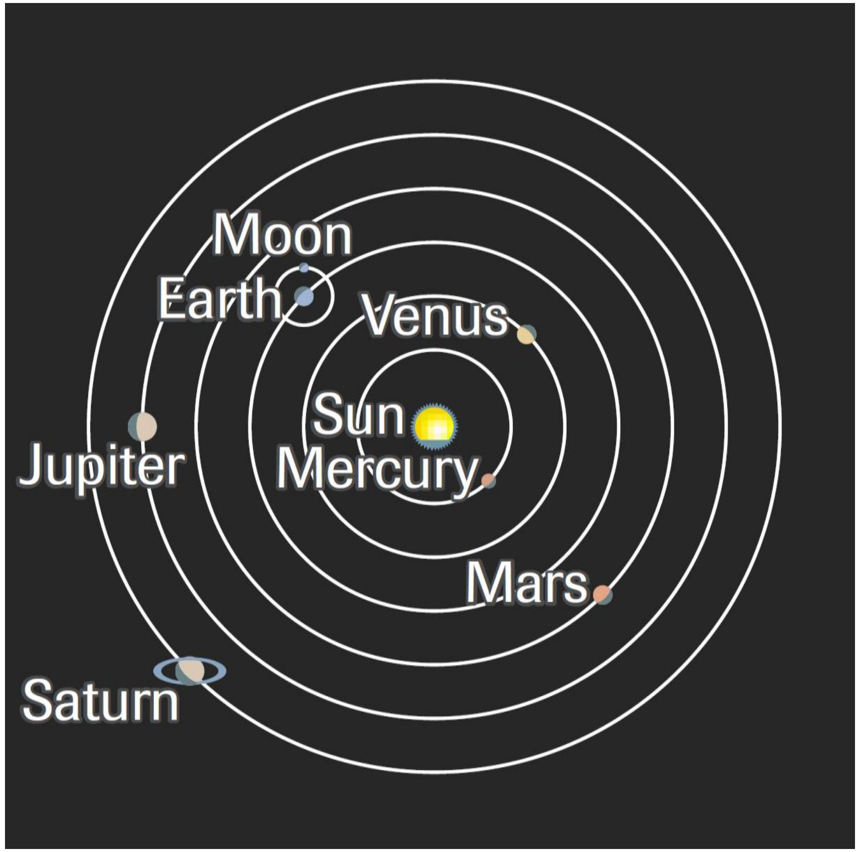
$$= \sqrt{\frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99 \times 10^{30}\ \text{kg})}{1.49 \times 10^{11}\ \text{m}}}$$

$$v_E = 2.98 \times 10^4\ \text{m/s}$$

Venus travels at a speed of 3.51×10^4 m/s and Earth travels more slowly at a speed of 2.98×10^4 m/s around the Sun.

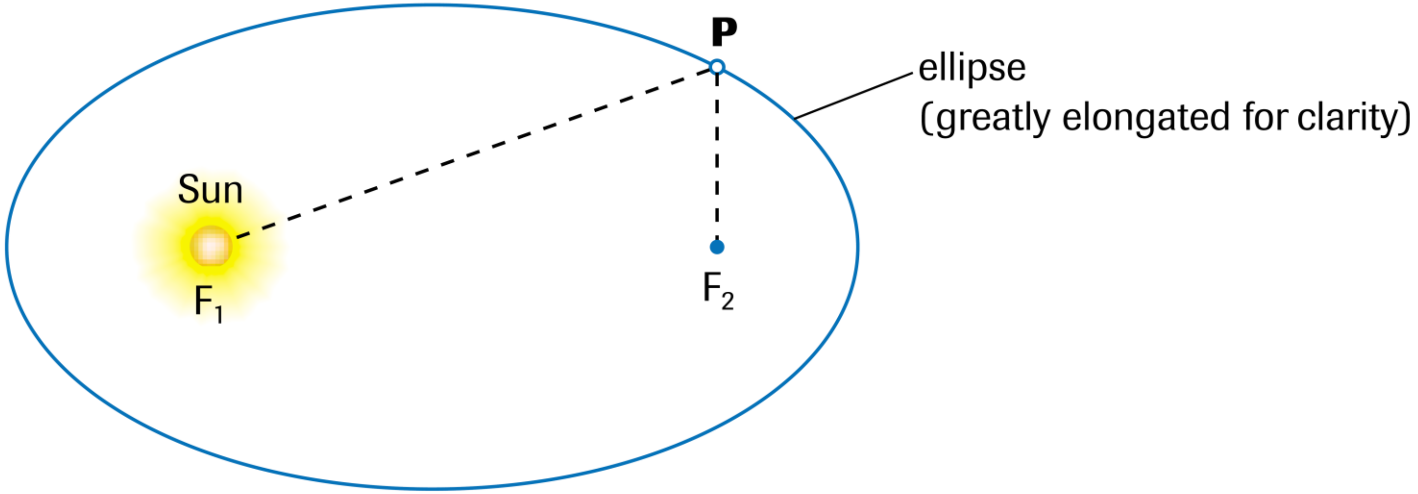
KEPLER'S LAWS





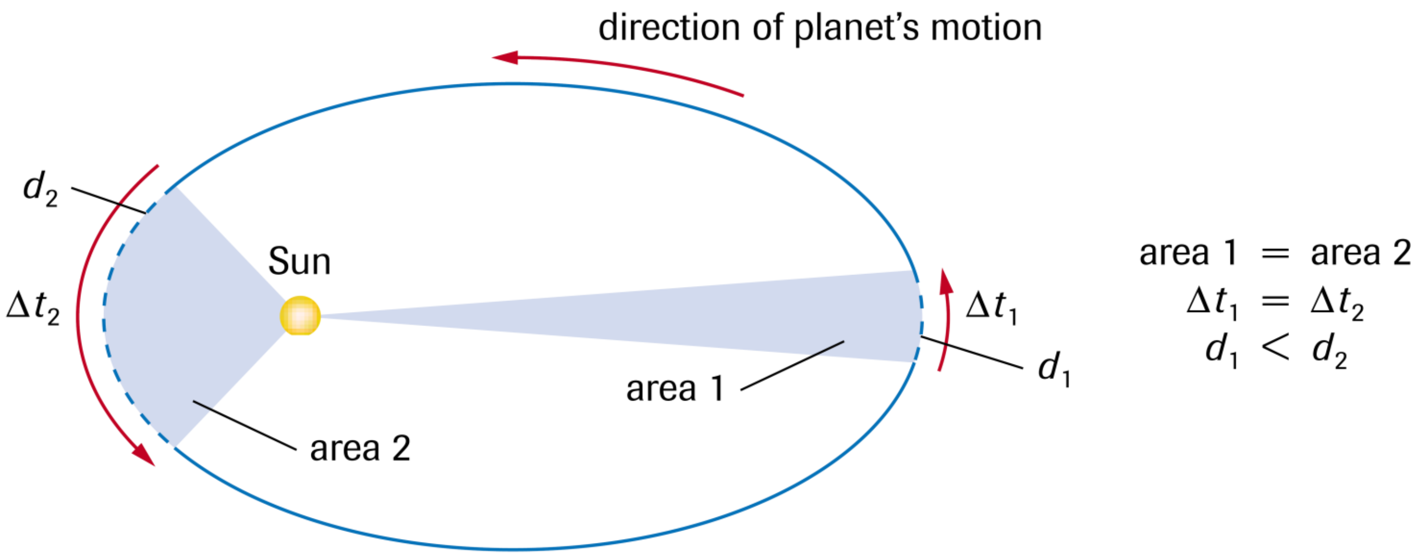
KEPLER'S 1ST LAW OF PLANETARY MOTION

- **Kepler's First Law of Planetary Motion:** Each planet moves around the Sun in an orbit that is an ellipse, with the Sun at one focus of the ellipse.



KEPLER'S 2ND LAW OF PLANETARY MOTION

- **Kepler's Second Law of Planetary Motion:** The straight line joining a planet and the Sun sweeps out equal areas in space in equal intervals of time.



KEPLER'S 3RD LAW OF PLANETARY MOTION

- **Kepler's Third Law of Planetary Motion:** The cube of the average radius r of a planet's orbit is directly proportional to the square of the period T of the planet's orbit.
- Mathematically, we get

$$r^3 \propto T^2$$
$$r^3 = C_S T^2$$
$$C_S = \frac{r^3}{T^2}$$

  - C_S – constant of proportionality for the Sun $[\mathrm{m^3/s^2}]$

PROBLEM 4

The average radius of orbit of Earth about the Sun is 1.495×10^8 km. The period of revolution is 365.26 days.

(a) Determine the constant C_S to four significant digits.

(b) An asteroid has a period of revolution around the Sun of 8.1×10^7 s. What is the average radius of its orbit?

PROBLEM 4 – SOLUTIONS

(a) $r_E = 1.495 \times 10^8 \text{ km} = 1.495 \times 10^{11} \text{ m}$

$T_E = 365.26 \text{ days} = 3.156 \times 10^7 \text{ s}$

$C_S = ?$

$$C_S = \frac{r^3}{T_E^2}$$

$$= \frac{(1.495 \times 10^{11} \text{ m})^3}{(3.156 \times 10^7 \text{ s})^2}$$

$$C_S = 3.355 \times 10^{18} \text{ m}^3/\text{s}^2$$

The Sun's constant is $3.355 \times 10^{18} \text{ m}^3/\text{s}^2$.

PROBLEM 4 – SOLUTIONS CONT.

(b) We can apply the Sun's constant found in (a) to this situation.

$C_S = 3.355 \times 10^{18}\ \text{m}^3/\text{s}^2$

$T = 8.1 \times 10^7\ \text{s}$

$r = ?$

$$\frac{r^3}{T^2} = C_S$$

$$r = \sqrt[3]{C_S T^2}$$

$$= \sqrt[3]{(3.355 \times 10^{18}\ \text{m}^3/\text{s}^2)(8.1 \times 10^7\ \text{s})^2}$$

$$r = 2.8 \times 10^{11}\ \text{m}$$

The average radius of the asteroid's orbit is 2.8×10^{11} m.

PROVING KEPLER'S 3RD LAW USING NEWTON'S LAW OF GRAVITATION

- We can equate the gravitational force of the Sun with its centripetal force

$$F_{G,S} = F_c$$

$$\frac{GM_S m_p}{r^2} = \frac{m_p v^2}{r}$$

- This simplifies to

$$v = \sqrt{\frac{GM_S}{r}}$$

PROVING KEPLER'S 3RD LAW USING NEWTON'S LAW OF GRAVITATION

- Since $T = \frac{2\pi r}{v}$, and $v = \sqrt{\frac{GM_S}{r}}$

$$T = \frac{2\pi r}{\sqrt{\frac{GM_S}{r}}}$$

- Which rearranges to

$$\frac{r^3}{T^2} = \frac{GM_S}{4\pi^2} = C_S$$

PROVING KEPLER'S 3RD LAW USING NEWTON'S LAW OF GRAVITATION

- In general, this can be written as

$$C = \frac{GM}{4\pi^2} = \frac{r^3}{T^2}$$

  - C – constant of proportionality for mass M
  - M – mass of the planetary body
  - r – distance of an object from the centre of mass
  - T – period of revolution for a satellite about mass M

KEPLER'S 3RD LAW APPLICATIONS

- Kepler's 3rd Law applies to any central body about which other bodies orbit
- IE: the Moon orbiting the Earth

$$C_E = \frac{GM_E}{4\pi^2} = \frac{r_M^3}{T_M^2}$$

SUMMARY – GRAVITATIONAL FIELDS

- A gravitational field exists in the space surrounding an object in which the force of gravity is exerted on objects.
- The magnitude of the gravitational field strength surrounding a planet or other body (assumed to be spherical) is directly proportional to the mass of the central body, and inversely proportional to the square of the distance to the centre of the body.
- The law of universal gravitation applies to all bodies in the solar system, from the Sun to planets, moons, and artificial satellites.

SUMMARY – ORBITS AND KEPLER'S LAWS

- The orbits of planets are most easily approximated as circles even though they are ellipses.
- Kepler's first law of planetary motion states that each planet moves around the Sun in an orbit that is an ellipse, with the Sun at one focus of the ellipse.
- Kepler's second law of planetary motion states that the straight line joining a planet and the Sun sweeps out equal areas in space in equal intervals of time.
- Kepler's third law of planetary motion states that the cube of the average radius r of a planet's orbit is directly proportional to the square of the period T of the planet's orbit.

PRACTICE

Readings

- Section 6.1 (pg 274)
- Section 6.2 (pg 278)

Questions

- pg 277 #1-4,6-8
- pg 284 #1-8